STRC201

ENGINEERING STRUCTURES

DATA SHEETS

Bending of Beams

Load, shear force and bending moment relationships:

$$w = -\frac{dV}{dx} \quad ; \quad V = \frac{dM}{dx}$$

Simple theory of bending:

$$\frac{M}{I} = \frac{E}{R} = \frac{-\sigma}{y}$$

Bending moment - curvature relationship:

$$EI\,\frac{d^2v}{dx^2} = M$$

Shear stress due to transverse loading:

$$\tau = \frac{V}{bI}\int_A ydA$$

Shear stress in thin open sections:

$$\tau_s = \frac{V}{tI}\int_o^s ydA$$

Bending nomenclature:

M	-	bending moment
V	-	shear force
w	-	loading/unit length
I	-	second moment of area about neutral axis
R	-	radius of curvature
E	-	modulus of elasticity
σ	-	stress
v	-	deflection of centroidal axis
τ	-	shear stress
τ_s	-	shear stress at position s
y	-	distance from NA
b	-	width of section at position y
t	-	thickness

(Over/...)

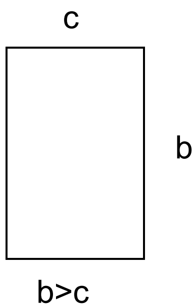
Torsion of Prismatic Bars

Simple torsion theory for circular cross-sections:

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$$

$$J = \frac{\pi d^4}{32}$$

Rectangular cross-sections



Stiffness $\frac{T}{\theta} = \frac{\beta b c^3 G}{L}$

Stress $\tau = \frac{T}{\alpha b c^2}$

b/c	1·0	1·5	1·75	2·0	2·5	3·0	4·0	6·0	10·0	∞
α	·208	·231	·239	·246	·258	·267	·282	·299	·313	·333
β	·141	·196	·214	·229	·249	·263	·281	·299	·313	·333

Thin-walled open sections.

The above results for rectangular cross-sections also apply to thin-walled open sections in which b is the developed length and c is the thickness of the cross-section.

Thin-walled closed sections:

Stiffness $\frac{T}{\theta} = \frac{4GA^2}{L\int \frac{dp}{t}}$

Stress $\tau = \frac{T}{2At}$

(Over/...)

Torsion nomenclature:

T	-	twisting moment
J	-	polar second moment of area
G	-	modulus of rigidity
θ	-	angular twist
L	-	length twisted
τ	-	shear stress
r	-	radius
d	-	diameter of bar
α β	-	constants for given sides ratio
p	-	developed length of cross-section
t	-	thickness
A	-	area enclosed by thin-walled closed section

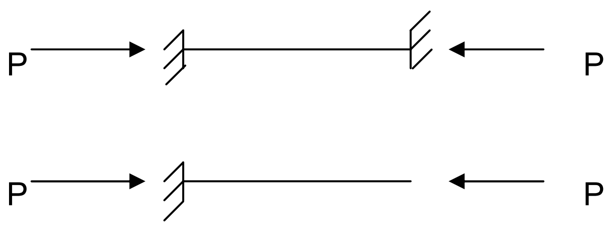
(Over/...)

Instability

Strut Theory

Euler theory

Geometry



Euler critical load (P_E)

$$P_E = \frac{\pi^2 EI}{L^2}$$

$$P_E = \frac{4\pi^2 EI}{L^2}$$

$$P_E = \frac{\pi^2 EI}{4L^2}$$

Rankine-Gordon formula

$$\sigma = \frac{\sigma_c}{1 + a\left(\frac{L}{k}\right)^2}$$

Instability nomenclature:

P_E	-	Euler buckling load
L	-	strut length
I	-	second moment of area about axis of bending
σ	-	stress $\left(\frac{P}{A}\right)$
k	-	radius of gyration $\left(\sqrt{\frac{I}{A}}\right)$
A	-	cross-sectional area
a	-	Rankine constant
σ_c	-	compressive yield stress

(Over/...)

Strain Energy

Strain energy U in terms of stress and stress resultants:

Uniaxial stress $U = \int \frac{\sigma^2}{2E}\, d(\text{volume})$

Shear stress $U = \int \frac{\tau^2}{2G}\, d(\text{volume})$

Bending $U = \int \frac{M^2 ds}{2EI}$

Torsion $U = \int \frac{T^2 ds}{2GJ}$

Energy methods:

Castigliano's theorems:

$$\delta_i = \frac{\partial U}{\partial W_i}$$

$$\theta_i = \frac{\partial U}{\partial M_i}$$

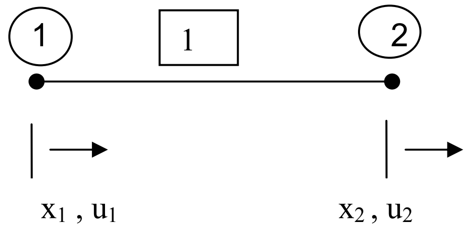
Nomenclature:

U	-	total strain energy
δ_i	-	deflection at i
W_i	-	load at i in direction i
θ_i	-	rotation at i
M_i	-	moment at i with same sense as θ_i

(Over/...)

Finite Element Stiffness Matrices

Simple tension/compression element:



Element force - displacement relationships:

$$\{X_i\} = [k_e]\{u_i\}$$

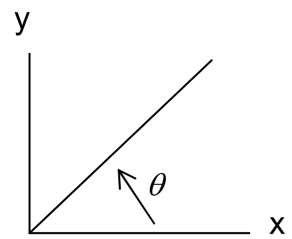
where $\{X_i\}$ is a vector of internal nodal forces = $\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$

where $\{u_i\}$ is a vector of nodal displacements = $\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

where $[k_e]$ is the element stiffness matrix = $\dfrac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Transformation matrix:

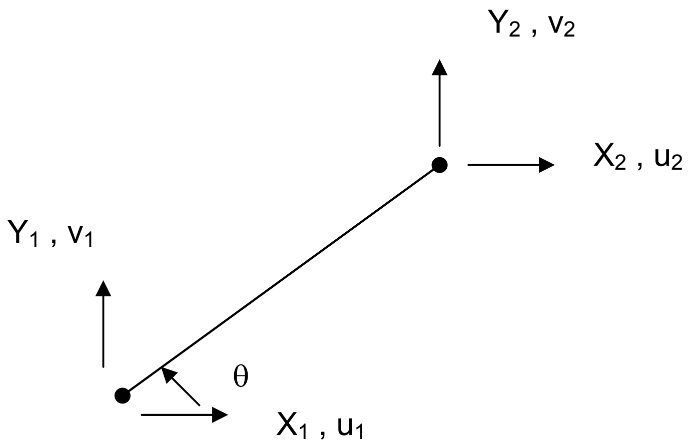
$$[t] = \begin{bmatrix} l & m \\ -m & l \end{bmatrix}$$



where $l = \cos\theta$
$m = \sin\theta$

(Over/...)

Two-dimensional element stiffness matrix for a simple tension/compression element:

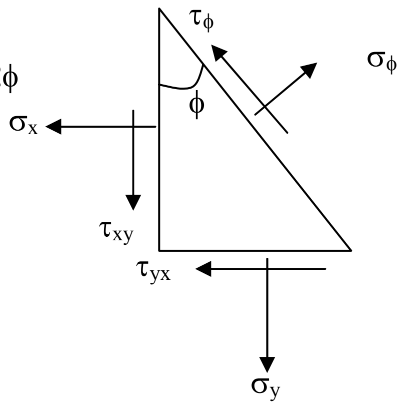


$$\begin{Bmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

(Over/...)

Two Dimensional Stress and Strain Transformation

Stresses on plane ϕ :



$$\sigma_\phi = \tfrac{1}{2}(\sigma_x + \sigma_y) + \tfrac{1}{2}(\sigma_x - \sigma_y)\cos 2\phi + \tau_{xy}\sin 2\phi$$

$$\tau_\phi = -\tfrac{1}{2}(\sigma_x - \sigma_y)\sin 2\phi + \tau_{xy}\cos 2\phi$$

Principal planes:

$$\text{Tan } 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Strains on plane θ :

$$\varepsilon_\theta = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) + \tfrac{1}{2}(\varepsilon_x - \varepsilon_y)\ \text{Cos } 2\theta + \tfrac{1}{2}\gamma_{xy}\sin 2\theta$$

$$\tfrac{1}{2}\gamma_\theta = -\tfrac{1}{2}(\varepsilon_x - \varepsilon_y)\sin 2\theta + \tfrac{1}{2}\gamma_{xy}\cos 2\theta$$

Stress - strain relationships for isotropic elastic materials :

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

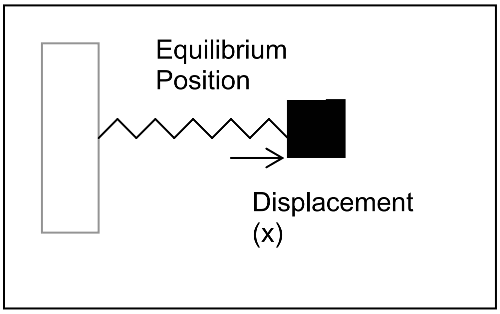
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

(Over/…..)

Mechanical Vibrations

Free Vibrations without Damping:



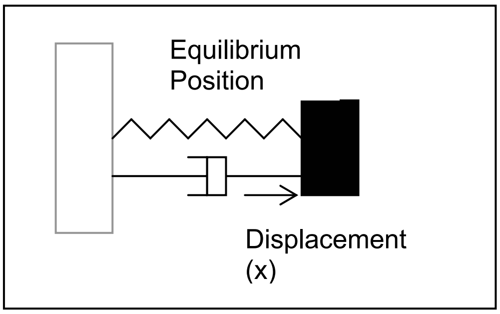
General equation of motion referred to the spring mass system:

$$\ddot{x} + \frac{k}{m}x = 0$$

Solution $x = A \cos \omega_n t + B \sin \omega_n t$

where A and B are constants and $\omega_n = \sqrt{\frac{k}{m}}$

Free Vibrations with Damping:



General equation of motion referred to the spring mass system:

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

Where:

m = Mass
k = Spring Stiffness
c = Viscous Damping Coefficient

There are three solutions to this equation depending on the magnitude of the damping. These are usually written in terms of the damping ratio (ζ) which is defined:

$\zeta = \frac{c}{2\omega_n m}$ and comprise:

(a) Under Damped $\zeta < 1$

$$x = e^{-\zeta\omega_n t} (A \cos \omega_d t + B \sin \omega_d t)$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

(b) Critically Damped $\zeta = 1$

$$x = (A + B)e^{-\zeta\omega_n t}$$

(Over/…..)

(c) Over Damped $\zeta > 1$

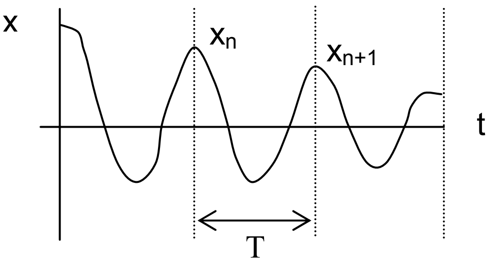
$$x = e^{-\zeta\omega_n t}\left(Ae^{\omega_n t\sqrt{\zeta^2 - 1}} + Be^{-\omega_n t\sqrt{\zeta^2 - 1}}\right)$$

Logarithmic Decrement (δ)

$$\delta = \text{Ln}\left(\frac{x_n}{x_{n+l}}\right) = \zeta\omega_n T$$

or $$\delta = \frac{2\pi\zeta}{\sqrt{(l-\zeta^2)}}$$

where T is the period $(T = \frac{2\pi}{\omega_d})$



Forced Vibration with Harmonic Excitation of Systems with Viscous Damping:

Forced vibration occurs when a system responds to an applied harmonic excitation. In general the response comprises a transient component and a steady state component. The former which is the complementary function solution to the differential equation of motion is excited when the applied vibration commences. The latter, the particular integral of the equation, is the steady state motion which persists after the transient vibration dies away due to system damping.

The transient motion is the same as the free vibration of the previous section and if under-damped, occurs with the damped frequency ω_d

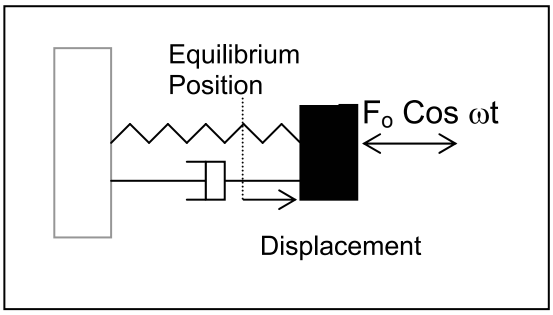
The steady state vibration occurs at the frequency of the harmonic excitation ω

Excitation of the Mass with a Harmonic Force of Amplitude F_0

General equation of motion

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m}\cos\omega t$$

where ω is the frequency of the harmonic excitation.



(Over/......)

General steady state solution

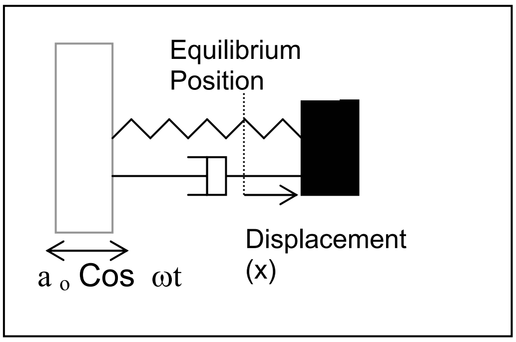
$$x = \frac{\dfrac{F_o}{k}}{\sqrt{\left(1-\left(\dfrac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\dfrac{\omega}{\omega_n}\right)^2}} \cos\left(\omega t + \phi\right)$$

Where the phase angle ϕ is given by:

$$\phi = \arctan \frac{2\zeta\dfrac{\omega}{\omega_n}}{1-\left(\dfrac{\omega}{\omega_n}\right)^2}$$

Excitation of the Support with a Harmonically Varying Displacement of Amplitude a_0

If the support vibrates harmonically with amplitude a_0 then the mass vibrates:



$$x = a_o \sqrt{\frac{1+\left(2\zeta\dfrac{\omega}{\omega_n}\right)^2}{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\dfrac{\omega}{\omega_n}\right)^2}} \cos\left(\omega t + \phi\right)$$

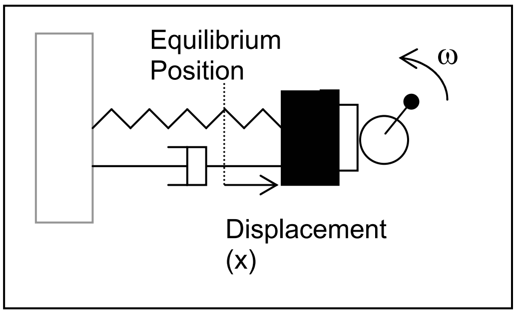
When the mass is excited by a harmonic vibration of the support structure then a reactive force is transmitted to the support. This force is given by:

$$F_T = kx_o\sqrt{1+\left(2\zeta\dfrac{\omega}{\omega_n}\right)^2}$$

(Over/…..)

Excitation of the Main Mass with a Rotating Out of Balance Mass m_e with an eccentricity r_e

If the eccentric mass rotates at angular velocity ω then:



$$x = \frac{m_e r_e}{m} \frac{(\omega/\omega_n)^2}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n{}^2}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}} \cos \quad (\omega t + \phi)$$

- END OF DATA BOOK -