Maximum and Minimum Values of Stress

The numerical variation of σ_ϕ and τ_ϕ for a given general stress state is given in Example 1. It can be seen that the maximum and minimum values of σ_ϕ occur at positions ~ 25^o and 145^o i.e. 90^o apart, and that the maximum and minimum values of τ_ϕ occur at positions ~ 45^o from the values of maximum σ_ϕ.

We can deduce these results mathematically.

Starting with equation (2):

$$s_f = \frac{1}{2}\left(s_x + s_y\right) + \frac{1}{2}\left(s_x - s_y\right)\cos 2f + t_x \sin 2f$$

The turning points are given by:

$$\frac{ds_f}{df} = 0$$

i.e.:

$$-\left(s_x - s_y\right)\sin 2f + 2t_x \cos 2f = 0$$

Hence, maximum and minimum values of σ_ϕ exist where:

$$\tan 2f = \frac{2t_x}{s_x - s_y} \qquad (4)$$

Equation (4) gives two value of ϕ denoting the PRINCIPAL PLANES.

Substitution of these two values of ϕ into equation (2) give the maximum and minimum values of σ_ϕ known as the PRINCIPAL STRESSES.

σ_1 = maximum value of σ_ϕ

σ_2 = minimum value of σ_ϕ

Starting with equation (3):

$$t_f = -\frac{1}{2}\left(s_x - s_y\right)\sin 2f + t_x \cos 2f$$

Equating to zero again gives equation (4):

$$\tan 2f = \frac{2t_x}{s_x - s_y}$$

Hence shear stress is zero on the principal planes.

It can also be shown that the maximum shear stress is given by:

$$t_{max} = \frac{1}{2}(s_1 - s_2) \qquad (5)$$

and occurs on planes +/- 45^o from principal planes.

Example 3

Determine analytically the directions and magnitudes of the principal stresses in Example 1. Hence find the maximum shear stress.

From equation (4), for the directions of σ_1 and σ_2:

$$\tan 2f = \frac{2t_x}{s_x - s_y} = \frac{-50}{20} = -2.5$$

Hence, 2ϕ = -68.2^o and 111.8^o

Substituting into equation (2):

$\sigma_{-34.1}$ = 20 + 10 cos (-68.2) –25 sin (-68.2)

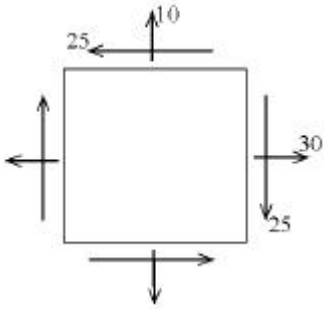
= 20 + 3.7 + 23.8

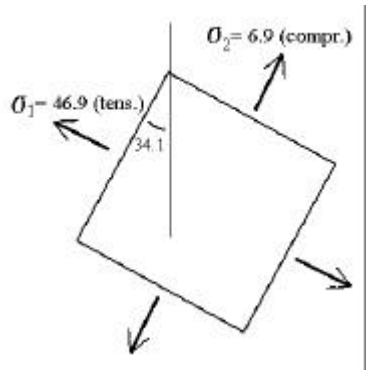
= 46.9

$\sigma_{55.9}$ = 20 + 10 cos (111.8) –25 sin (111.8)

= 20 - 3.7 - 23.8

= -6.9





Maximum shear stress on planes at 45^o to principal planes (ϕ = -79.1^o and 10.9^o).

$\tau_{-79.1}$ = -10 sin (-158.2) –25 cos (-158.2)

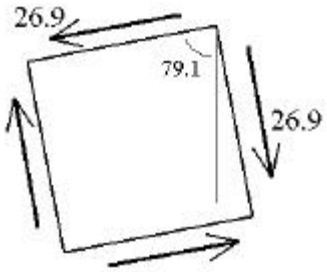
= 3.7 + 23.2

= 26.9

$\tau_{10.9}$ = -10 sin (21.8) –25 cos (21.8)

= -3.7 - 23.2

= -26.9



Alternatively:

$$t_{max} = \frac{1}{2}(s_1 - s_2)$$

= 0.5 (46.9 + 6.9)

= 26.9

Shear stresses are towards σ_1 (away from σ_2).