Two Dimensional Stress Transformation

A set of equations known as the Stress Transformation Equations may be used to calculate the normal and shear stresses on any plane through a point where the state of stress is defined (by σ_x, σ_y and τ_x). It is often more convenient however not to use these equations directly, but to employ a graphical method of solution known as Mohr's Stress Circle. Although it is hoped that the reader will follow through the derivation of the Stress Transformation Equations presented in the appendix to this chapter, the emphasis of this course will be placed on Mohr's methods of solution.

You should be familiar with the concept of stress to describe the transmission of internal forces within a body. Thus on any internal plane we may identify two types of stress: a direct stress (perpendicular to the plane) and a shear stress (tangential to the plane).

If we consider an internal plane inclined at some angle ϕ to the direction of loading, it is clear that both normal and tangential forces will exist on the plane to maintain equilibrium. Therefore a system of direct and shear stresses will exist on the inclined plane.

Consider the bar shown in Figure 1 subjected to an axial force F through the centroid of the cross section. A rectangular element ABCD within the bar will be subjected to direct stresses (Force/Area) $\sigma_x = F/(AD)t$ on faces AD and BC of the element.

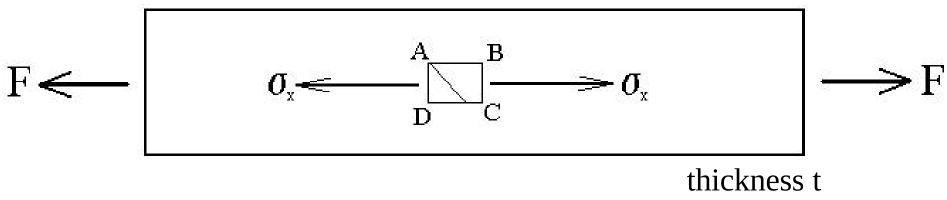

Figure 1

Consider now the stresses on plane AN, inclined at angle ϕ to AD. For equilibrium of the triangular element to be maintained, a direct stress σ_ϕ and a shear stress τ_ϕ will be required to act on the plane as shown in Figure 2.

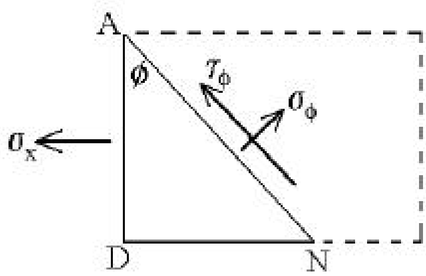

Figure 2

The Stress Transformation equations can be used to calculate the normal and shear stresses on any such plane through a point where the state of stress is defined (by σ_x,

σ_y and τ_x). The stresses on this inclined plane can be determined by a graphical construction. This construction, known as Mohr's Stress Circle was developed by the German engineer Otto Mohr.

Mohr's Circle

Rewriting the Stress Transformation equations:

$$\sigma_{\phi} - \frac{1}{2}\left(\sigma_x + \sigma_y\right) = \frac{1}{2}\left(\sigma_x - \sigma_y\right)\cos 2\phi + \tau_x \sin 2\phi$$

and:

$$\tau_{\phi} = -\frac{1}{2}\left(\sigma_x - \sigma_y\right)\sin 2\phi + \tau_x \cos 2\phi$$

Squaring both sides and adding:

$$\left[\sigma_{\phi} - \frac{1}{2}\left(\sigma_x - \sigma_y\right)\right]^2 + \tau_x^2 = \frac{1}{4}\left(\sigma_x - \sigma_y\right)^2 + \tau_x^2$$

We may compare this with the equation of a circle, thus:

$$x^2 + y^2 = r^2$$

The radius of this circle is therefore:

$$\left\{\frac{1}{4}\left(\sigma_x - \sigma_y\right)^2 + \tau_x^2\right\}^{\frac{1}{2}}$$

And the centre has the co-ordinates:

$$\frac{1}{2}\left(\sigma_x + \sigma_y\right),\ 0$$

This circle represents the state of stress at the point considered and gives the normal and shear stress on any plane through the point.

The stressed element and corresponding Mohr's stress circle are shown in Figure 3.

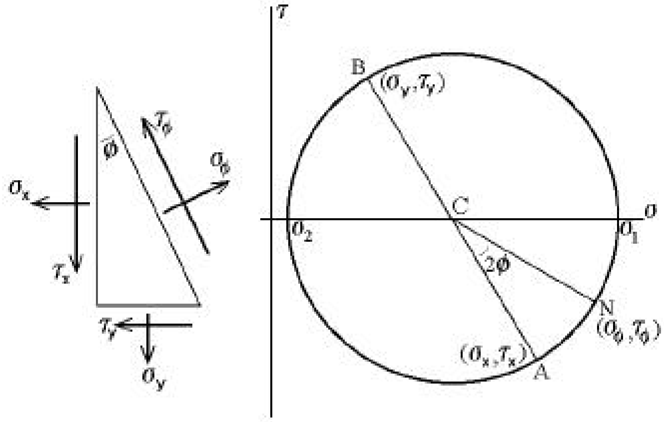

Figure 3.

<u>Sign Convention</u>

Positive normal stresses are plotted to the right of the origin, negative normal stresses are plotted to the left.

Clockwise shear stress components are plotted above the abscissa, anticlockwise shear stresses below.

<u>Construction of the Stress Circle</u>

Set up the co-ordinate axes to represent normal and shear stress to the same scale.

Plot points A (σ_x , τ_x) and B (σ_y , τ_y) representing the direct and shear stresses on the two faces of the element.

The line joining these two points is a diameter of the stress circle. The intersection of this diameter and the normal stress axis is the centre of the circle C, which should now be plotted.

An angle of 2ϕ is set off from the line CA in an anticlockwise direction (representing an anticlockwise rotation ϕ in the element) and is labelled CN. The co-ordinates of point N represent the normal and shear stresses on the inclined plane.

<u>Three-Dimensional Problems</u>

The Mohr's circle technique may be extended and applied to three-dimensional problems. However, since the vast majority of the problems encountered by the mechanical engineer are either two-dimensional, or may be considered to be two-dimensional, the application of Mohr's methods to three-dimensions is considered to be of rare value and is not covered in this text.