Two Dimensional Strain Transformation

This section is intended as background reading to accompany the notes on Mohr's Strain Circle and contains a rigorous derivation of the Strain Transformation Equations and an introduction to strain gauge rosettes.

Introduction

For any given two dimensional stress system a deformable body will undergo deformation. In many engineering problems the displacements will be small. This assumption will form the basis of the following analysis.

General two Dimensional Strain System

As for the stress problem we first define a general two-dimensional deformation system. The sign convention adopted may be arbitrary however in this treatment it is deliberately chosen to give consistency with the adopted stress sign convention and with the theory of elasticity.

The deformed shape is given in Figure 1.

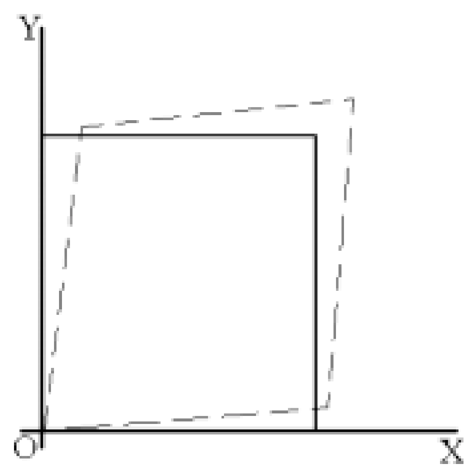

Figure 1.

Sign convention:

- ε_x – extensional strain +ve
- ε_y - extensional strain +ve
- γ_{xy} – decrease in right-angle XOY is +ve

Note: Rigid body movement at and about 'O' is not considered.

Complementary Shear Strain

If we consider an X-Y system initially drawn on an undeformed plate which becomes X^1 -Y^1 after deformation we observe (Figure 2):

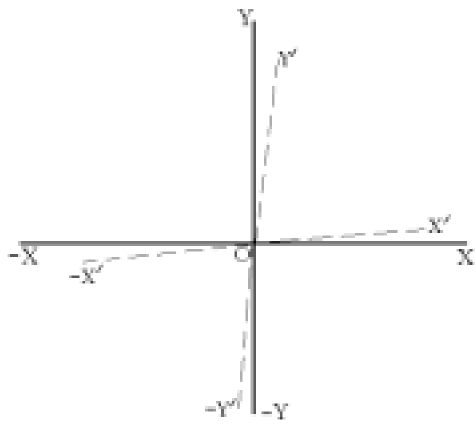

Figure 2

Note:

γ_{xy} is +ve
γ_{yx}* is –ve and numerically equal to γ_{xy} i.e. $\gamma_{yx} = - \gamma_{xy}$

* change in right angle YOX

Transformation Equations

Consider the strain components in the XOY axes shown in Figure 3.

We wish to determine the strain components in the aOb axes set shown in which Oa makes angle θ with OX.

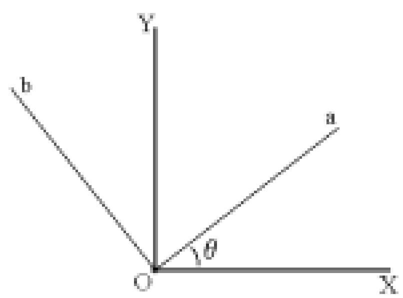

Figure 3.

The problem may be analysed geometrically to determine the transformation equations and as might be expected for linear elastic material the strain transformation equations are of similar form to the stress transformation equations.

The results are as follows:

$$\varepsilon_a = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta \tag{1}$$

$$\frac{\gamma_{ab}}{2} = -\frac{1}{2}(\varepsilon_x - \varepsilon_y)\sin 2\theta + \frac{1}{2}\gamma_{xy}\cos 2\theta \tag{2}$$

It can be seen by inspection that the above Strain Transformation Equations are identical to the Stress Transformation Equations if the equivalence is made that:

$$\varepsilon \equiv \sigma$$

$$\frac{\gamma}{2} \equiv \tau$$

It follows that the variation of ε_a and $\gamma_{ab}/2$ with θ will be similar to that of σ_ϕ and τ_ϕ with ϕ.

Also, the principal strains ε_1 and ε_2 will occur on principal planes defined by:

$$\tan 2\theta = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \tag{3}$$