

ii) Principal planes defined by ϕ

$$\text{where } \tan 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\therefore \tan 2\phi = \frac{2(-20)}{40 - 30} = -4$$

$$2\phi = -75.96^\circ \text{ and } 104.04^\circ$$

$$\therefore \phi = -37.98^\circ \text{ and } 52.02^\circ$$

$$\text{Say } -38^\circ \text{ and } 52^\circ$$

Substituting these values of ϕ into the stress transformation equation for σ_ϕ will give the principal stresses.

$$\begin{aligned}\sigma_{-38^\circ} &= \frac{1}{2}(40 + 30) + \frac{1}{2}(40 - 30)\cos(-76^\circ) - 20\sin(-76^\circ) \\ &= 35 + 1.2 + 19.4 = \underline{55.6 \text{ MPa}}\end{aligned}$$

$$\begin{aligned}\sigma_{52^\circ} &= \frac{1}{2}(40 + 30) + \frac{1}{2}(40 - 30)\cos 104^\circ - 20\sin 104^\circ \\ &= 35 - 1.2 - 19.4 = \underline{14.4 \text{ MPa}}\end{aligned}$$

Principal stresses are :

$$\sigma_1 = 55.6 \text{ MPa}$$

$$\sigma_2 = 14.4 \text{ MPa}$$