

## Engineering Science - MECH 226 - Formula Sheet

### Statics

**Beams** - Load, shear force and bending moment relationships:

$$w = -\frac{dV}{dx} ; \quad V = \frac{dM}{dx}$$

**Simple theory of bending:**  $\frac{M}{I} = \frac{E}{R} = \frac{-\sigma}{y}$

**Bending moment - curvature relationship:**  $EI \frac{d^2v}{dx^2} = M$

**Simple theory of torsion for circular a section:**  $\frac{\tau}{r} = \frac{T}{J} = \frac{G\phi}{L} ; \quad J = \frac{\pi d^4}{32}$

**For a slender column with thin ends:**  $P_{cr} = \frac{\pi^2 EI}{L^2}$

**Thin cylinders:**  $\sigma_{\theta} = \frac{pd}{2t} ; \quad \sigma_l = \frac{pd}{4t} ;$  **Thin sphere:**  $\sigma_{\theta} = \frac{pd}{4t}$

**Two dimensional stress and strain** - relationships for isotropic elastic materials:

$$\gamma_{xy} = \frac{\tau_{xy}}{G} ; \quad \epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) ; \quad \sigma_x = \frac{E}{1-\nu^2}(\epsilon_x + \nu\epsilon_y) ; \quad E = 2G(1+\nu)$$

**Two dimensional stress and strain transformations** - stresses on plane at  $\phi$

$$\sigma_{\phi} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\phi + \tau_{xy}\sin 2\phi$$

$$\tau_{\phi} = -\frac{1}{2}(\sigma_x - \sigma_y) \sin 2\phi + \tau_{xy} \cos 2\phi$$

Principal planes:  $\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

Strains on plane at  $\theta$

$$\epsilon_{\theta} = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y) \cos 2\theta + \frac{1}{2}\gamma_{xy} \sin 2\theta$$

$$\frac{1}{2}\gamma_{\theta} = -\frac{1}{2}(\epsilon_x - \epsilon_y) \sin 2\theta + \frac{1}{2}\gamma_{xy} \cos 2\theta$$

**Dynamics** - acceleration:  $F = Ma$  ;  $T = I\alpha$

**Moments of inertia:**

for a thin disc, polar axis:  $I_p = \frac{Mr^2}{2}$  ; about diameter:  $I_{dia} = \frac{Mr^2}{4}$

for a slender rod about its centre of mass:  $I = \frac{ml^2}{12}$

**Equations of motion** - for constant acceleration:  $v = u + at$  ;

$$v^2 = u^2 + 2as$$
 ;  $s = ut + \frac{1}{2}at^2$

**Acceleration of a geared system** - torque on A to accelerate A and B:

$$T_A = (I_A + n^2 I_B) \text{ where } n = \frac{\omega_B}{\omega_A}$$

## Vibration

**Free undamped vibration** - general equation:  $x'' + \frac{k}{m}x = 0$  the general solution is:

$$x = \frac{x'(0)}{\omega_n} \sin \omega_n t + x(0) \cos \omega_n t \quad \text{where } \omega_n = \sqrt{\frac{k}{m}}$$

**Free vibration with damping** - general equation:  $x'' + \frac{c}{m}x' + \frac{k}{m}x = 0$

where  $m$  = mass,  $k$  = spring stiffness and  $c$  = viscous damping coefficient.

There are three solutions to this equation depending upon the magnitude of the damping. These are usually written in terms of the damping ratio  $\zeta$  which is defined as:

$$\zeta = \frac{c}{2\omega_n m} \quad \text{The three solutions are:}$$

a) Under damped  $\zeta < 1$ :  $x = e^{-\zeta\omega_n t} (A \cos \omega_d t + B \sin \omega_d t)$

where  $\omega_d = \omega_n \sqrt{1 - (\zeta)^2}$

b) Critically damped  $\zeta = 1$ :  $x = (A + Bt)e^{-\zeta\omega_n t}$

c) Over damped  $\zeta > 1$ :  $x = e^{-\zeta\omega_n t} (Ae^{\omega_n t \sqrt{\zeta^2 - 1}} + Be^{-\omega_n t \sqrt{\zeta^2 - 1}})$

$$\text{Logarithmic decrement: } \delta = \ln \left( \frac{x_n}{x_{n+1}} \right) = \zeta \omega_n T = \frac{2\pi\zeta}{\sqrt{1 - \zeta^2}}$$

where  $T$  is the periodic time:  $T = \frac{2\pi}{\omega_d}$